

## Relazione finale Progetto Marco Polo di Lorenzo Zanelli

The project has involved the study of semiclassical approximation in quantum systems, in particular by the application of central results of weak KAM Theory and Aubry-Mather Theory in this framework.

More precisely, I have focused the attention on the spectral study and on the study of quasimodes for a class of Schrödinger operators on the torus. These objectives have been obtained by the use of the so called weak KAM solutions of the Hamilton-Jacobi equation as well as by the use of the solutions of the coupled continuity equation written in the measure sense.

In particular, my results have exhibited a direct link between a meaningful class of energy quasimodes (naturally generalizing the which ones coming from the usual WKB technique) and the important subclass of Mather measures (i.e. invariant probability measures for the classical Lagrangian flow on the phase space and that fulfill the principle of minimal Action) which are arising from the continuity equation on the torus. Moreover, I have also proved in detail the relationship between the asymptotic classical limit of the mean value of non commuting quantum observables on these energy quasimodes and the so-called Aubry sets.

As a consequence, I have also solved a Conjecture stated by L.C. Evans (Towards a quantum analog of weak KAM theory. *Comm. Math. Phys.* 244, 2004, no. 2, 311–334) about some properties of a class of energy quasimodes in the general setting of mechanical Hamiltonians on the torus without assumption of integrability or quasi-integrability.

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